NAG Toolbox for MATLAB

f08cv

1 Purpose

f08cv computes an RQ factorization of a complex m by n matrix A.

2 Syntax

```
[a, tau, info] = f08cv(a, 'm', m, 'n', n)
```

3 Description

f08cv forms the RQ factorization of an arbitrary rectangular real m by n matrix. If $m \leq n$, the factorization is given by

$$A = \begin{pmatrix} 0 & R \end{pmatrix} Q,$$

where R is an m by m lower triangular matrix and Q is an n by n unitary matrix. If $m > n$ the factorization is given by

$$A = RQ,$$

where R is an m by n upper trapezoidal matrix and Q is again an n by n unitary matrix. In the case where $m < n$ the factorization can be expressed as

$$A = \begin{pmatrix} 0 & R \end{pmatrix} \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = RQ_2,$$

where Q_1 consists of the first $(n - m)$ rows of Q and Q_2 the remaining m rows.

The matrix Q is not formed explicitly, but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,∗) – complex array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \leq n$, the upper triangle of the subarray $\mathbf{a}(1 : m, n - m + 1 : n)$ contains the m by m upper triangular matrix R.

If $m \geq n$, the elements on and above the $(m - n)$th subdiagonal contain the m by n upper trapezoidal matrix R; the remaining elements, with the array **tau**, represent the unitary matrix Q as a product of $\min(m, n)$ elementary reflectors (see Section 3.2.6 in the F08 Chapter Introduction).

2: **tau(∗) – complex array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

The scalar factors of the elementary reflectors.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $A + E$, where

$$\|E\|_2 = O\epsilon\|A\|_2$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating point operations is approximately $\frac{2}{3}m^2(3n-m)$ if $m \leq n$, or $\frac{2}{3}n^2(3m-n)$ if $m > n$.

To form the unitary matrix Q f08cv may be followed by a call to f08cw:

```
CALL ZUNGRQ (N,N,MIN(M,N),A,LDA,TAU,WORK,LWORK,INFO)
```

```
[a, info] = f08cw(a, tau, 'k', min(m,n));
```

but note that the first dimension of the array **a** must be at least **n**, which may be larger than was required by f08cv. When $m \leq n$, it is often only the first m rows of Q that are required and they may be formed by the call:

```
CALL ZUNGRQ (M,N,M,A,LDA,TAU,WORK,LWORK,INFO)
```

```
[a, info] = f08cw(a, tau);
```

To apply Q to an arbitrary real rectangular matrix C, f08cv may be followed by a call to f08cx. For example:

```
[a, c, info] = f08cx('Left','C', a, tau, c);  forms C = Q^T C, where C is n by p.
```

The real analogue of this function is Missing 'id'.

9 Example

```
a = [complex(0.28, -0.36), complex(0.5, -0.86), complex(-0.77, -0.48), complex(1.58, +0.66);
       complex(-0.5, -1.1), complex(-1.21, +0.76), complex(-0.32, -0.24), complex(-0.27, -1.15);
      complex(0.36, -0.51), complex(-0.07000000000000001, +1.33), complex(-0.75, +0.47), complex(-0.08, +1.01)];
[aOut, tau, info] = f08cv(a)  aOut =
  -0.4691 + 0.3700i  -1.4356           -0.5819 + 1.4047i  -0.0012 - 0.7617i
   0.2378 + 0.7550i   0.1432 - 0.4602i   2.0877            0.2327 + 0.8028i
  -0.2372 + 0.1304i   0.2798 - 0.5052i   0.3816 - 0.0408i   1.9933
tau =
   1.3820 + 0.3564i
   1.0232 + 0.2322i
   1.0401 + 0.5067i
info =
           0
```